16 mit f

$$f(x) = \frac{11x-1}{(3x-1)(x+1)} = \frac{A}{3x-1} + \frac{B}{x+1} \quad \Big| \cdot (3x-1)(x+1)$$

$$11x-1 = A(x+1) + B(3x-1)$$

$$11x-1 = Ax + A + 3Bx - B$$

$$11x-1 = (A+3B)x + A - B$$

$$\text{I} \quad A + 3B = 11$$

$$\text{II} \quad A - B = -1$$

$$\text{I}-\text{II} \quad 4B = 12$$

$$B = 3$$

$$A - 3 = -1$$

$$A = 2$$

$$\int \frac{11x-1}{(3x-1)(x+1)}\,dx = \int \frac{2}{3x-1}\,dx + \int \frac{3}{x+1}\,dx$$

$$= \frac{2}{3}\ln(|3x-1|) + 3\ln(|x+1|) + C$$

oder

$$\frac{11x-1}{(3x-1)(x+1)} = \frac{A}{3x-1} + \frac{B}{x+1} \quad \Big|\cdot(3x-1) \cdot(x+1)$$

$$11x-1 = A(x+1) + B(3x-1)$$

$x = -1$ einsetzen

$$-12 = A \cdot 0 + B(-4)$$

$$3 = B$$

$x = \frac{1}{3}$ einsetzen

$$\frac{11}{3} - 1 = A\left(\frac{1}{3}+1\right) + B \cdot 0$$

$$\frac{8}{3} = \frac{4}{3}A$$

$$2 = A$$

$$\Rightarrow \frac{11x-1}{(3x-1)(x+1)} = \frac{2}{3x-1} + \frac{3}{x+1}$$

$$\int \frac{2}{3x-1}\,dx + \int \frac{3}{x+1}\,dx = \frac{2}{3}\ln(|3x-1|) + 3\ln(|x+1|) + C$$

16 mit g

$$g(x) = \frac{22x+7}{(x+1)(4x+1)} = \frac{A}{x+1} + \frac{B}{4x+1}$$

$$22x+7 = A(4x+1) + B(x+1)$$

$x = -1$ einsetzen

$$-15 = -3A$$

$$A = 5$$

$x = -\frac{1}{4}$ einsetze

$$-\frac{22}{4} + 7 = B\left(-\frac{1}{4}+1\right)$$

$$\frac{6}{4} = \frac{3}{4}B$$

$$B = 2$$

$$\int \frac{22x+7}{(x+1)(4x+1)}\,dx = \int \frac{5}{x+1}\,dx + \int \frac{2}{4x+1}\,dx$$

$$= 5\ln(|x+1|) + \frac{2}{4}\ln(|4x+1|) + C$$

$$= 5\ln(|x+1|) + \frac{1}{2}\ln(|4x+1|) + C$$

16 mit h

$$h(x) = \frac{10x-4}{x(2x-1)} = \frac{A}{x} + \frac{B}{2x-1}$$

$$10x-4 = A(2x-1)+Bx$$

$x=0$ einsetze

$$-4 = -A$$

$$A = 4$$

$x=\frac{1}{2}$ einsetze

$$5-4 = \frac{1}{2}B$$

$$B = 2$$

$$\int \frac{10x-4}{x(2x-1)}\,dx = \int \frac{4}{x}\,dx + \int \frac{2}{2x-1}\,dx$$

$$= 4\ln(|x|) + \ln(|2x-1|) + C$$

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$$\frac{6x^2-5x+2}{(x+2)(x-1)^2} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$6x^2-5x+2 = A(x-1)^2 + B(x-1)(x+2) + C(x+2)$$

$x=1$ einsetzen

$$6-5+2 = 3C$$

$$1 = C$$

$x=-2$ einsetze

$$24+10+2 = 9A$$

$$4 = A$$

$x=0 \quad A=4 \quad C=1$

$$2 = 4 - 2B + 2\cdot 1$$

$$-4 = -2B$$

$$2 = B$$

$$\frac{6x^2-5x+2}{(x+2)(x-1)^2} = \frac{4}{x+2} + \frac{2}{x-1} + \frac{1}{(x-1)^2}$$

$$\int f(x)\,dx = \int \frac{4}{x+2}\,dx + \int \frac{2}{x-1}\,dx + \int (x-1)^{-2}\,dx$$

$$= 4\ln(|x+2|) + 2\ln(|x-1|) - (x-1)^{-1} + C$$

oder

$$6x^2-5x+2 = A(x-1)^2 + B(x-1)(x+2) + C(x+2)$$

$$6x^2-5x+2 = Ax^2-2Ax+A+Bx^2+Bx-2B+Cx+2C$$

$$6x^2-5x+2 = (A+B)x^2 + (-2A+B+C)x + A-2B+2C$$

$$A+B = 6$$

$$-2A+B+C = -5 \quad | \cdot 2$$

$$A-2B+2C = 2$$

ohne Trick mit Gleichungs-System

$$-4A+2B+2C = -10$$

$$A-2B+2C = 2$$

$$-5A+4B = -12$$

$$A+B = 6 \quad | \cdot 4$$

$$-5A+4B = -12$$

$$4A+4B = 24$$

$$-9A = -36$$

$$A = 4$$

$$4+B = 6 \Rightarrow B = 2$$

$$4-2\cdot 2+2C = 2$$

$$C = 1$$

$$\frac{6x^2-5x+2}{(x+2)(x-1)^2} = \frac{A}{x+2} + \frac{Bx+\gamma}{(x-1)^2}$$

$$6x^2-5x+2 = A(x-1)^2 + (Bx+\gamma)(x+2)$$

$$6x^2-5x+2 = Ax^2 - 2Ax + A + Bx^2 + 2Bx + \gamma x + 2\gamma$$

$$6x^2-5x+2 = (A+B)x^2 + (-2A+2B+\gamma)x + (A+2\gamma)$$

$$\begin{aligned} A + B &= 6 \quad &| \cdot 2 \\ -2A + 2B + \gamma &= -5 \quad &| \cdot 1 \\ A + 2\gamma &= 2 \end{aligned}$$

$$\begin{aligned} 2A + 2B &= 12 \\ -2A + 2B + \gamma &= -5 \end{aligned}$$

$$\begin{aligned} 4A - \gamma &= 17 \quad | \cdot 2 \\ A + 2\gamma &= 2 \end{aligned}$$

$$\begin{aligned} 8A - 2\gamma &= 34 \\ A + 2\gamma &= 2 \end{aligned}$$

$$9A = 36$$

$$A = 4$$

$$4 + 2\gamma = 2 \qquad 4 + B = 6$$

$$\gamma = -1 \qquad B = 2$$

$$\frac{6x^2-5x+2}{(x+2)(x-1)^2} = \frac{4}{x+2} + \frac{2x-1}{(x-1)^2}$$

$$\int \frac{6x^2-5x+2}{(x+2)(x-1)^2}\,dx = \int \frac{4}{x+2}\,dx + \int \frac{2x-1}{x^2-2x+1}\,dx$$

$$= 4\ln(|x+2|) + \int \frac{2x-1}{x^2-2x+1}\,dx$$

$$= 4\ln(|x+2|) + \int \frac{2x-1-1+1}{x^2-2x+1}\,dx$$

$$= 4\ln(|x+2|) + \int \frac{2x-2+1}{x^2-2x+1}\,dx$$

$$= 4\ln(|x+2|) + \int \frac{2x-2}{x^2-2x+1}\,dx + \int \frac{1}{x^2-2x+1}\,dx$$

↓ (*)

$$= 4\ln(|x+2|) + 2\ln(|x-1|) + \int (x-1)^{-2}\,dx$$

$$= 4\ln(|x+2|) + 2\ln(|x-1|) - (x-1)^2 + C$$

(*) $$\int \frac{2x-2}{x^2-2x+1}\,dx = \ln(|x^2-2x+1|) = \ln((x-1)^2)$$
$$= 2\ln(|x-1|)$$

oder

$$\int \frac{2x-2}{x^2-2x+1}\,dx = \int \frac{2(x-1)}{(x-1)^2}\,dx$$
$$= \int \frac{2}{(x-1)}\,dx = 2\ln(|x-1|)$$

I $\dfrac{6x^2-5x+2}{(x+2)(x-1)^2} = \dfrac{A}{x+2} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$

Warum nicht

II $\dfrac{6x^2-5x+2}{(x+2)(x^2-2x+1)} = \dfrac{A}{x+2} + \dfrac{\beta x+\gamma}{x^2-2x+1}$

I $\dfrac{6x^2-5x+2}{(x+2)(x-1)^2} = \dfrac{4}{x+2} + \dfrac{2}{x-1} + \dfrac{1}{(x-1)^2}$

II $\dfrac{6x^2-5x+2}{(x+2)(x^2-2x+1)} = \dfrac{4}{x+2} + \dfrac{2x-1}{x^2-2x+1}$

$$\frac{2(x-1)}{(x-1)^2} + \frac{1}{(x-1)^2} = \frac{2x-1}{(x-1)^2}$$

oder

$$\frac{2x-1}{x^2-2x+1} = \frac{2x-1}{(x-1)^2} = \frac{2x-2+1}{(x-1)^2}$$

$$= \frac{2(x-1)+1}{(x-1)^2} = \frac{2\cancel{(x-1)}}{(x-1)^{\cancel{2}}} + \frac{1}{(x-1)^2}$$