17c

$$\frac{5x^2-24x+36}{x(x-3)^2} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$$5x^2-24x+36 = A(x-3)^2 + B(x-3)x + Cx$$

$x=3$ einsetzen

$$5\cdot 9 - 24\cdot 3 + 36 = 3C$$

$$9 = 3C \rightsquigarrow C=3$$

$x=0$ einsetzen

$$36 = A\,9 \rightsquigarrow A=4$$

$C=3 \quad A=4 \quad x=1$

$$5-24+36 = 4\cdot(-2)^2 + B(-2)\cdot 1 + 3\cdot 1$$

$$17 = 16 - 2B + 3$$

$$-2 = -2B \rightsquigarrow B=1$$

$$\int f(x)\,dx = \int \frac{4}{x}\,dx + \int \frac{1}{x-3}\,dx + \int \frac{3}{(x-3)^2}\,dx$$

$$= 4\ln(|x|) + \ln(|x-3|) + \int 3(x-3)^{-2}\,dx$$

$$= 4\ln(|x|) + \ln(|x-3|) - 3(x-1)^{-1} + C$$

18a)

$$\int \frac{2x^2+5x+2}{x^3+x}\,dx$$

$$\frac{2x^2+5x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$2x^2+5x+2 = A(x^2+1) + (Bx+C)x$$

$$2x^2+5x+2 = Ax^2+A+Bx^2+Cx$$

$$2x^2+5x+2 = (A+B)x^2+Cx+A$$

$$A+B=2$$

$$C=5$$

$$A=2 \Rightarrow 2+B=2 \Rightarrow B=0$$

$$\int \frac{2x^2+5x+2}{x^3+x}\,dx = \int \frac{2}{x}\,dx + \int \frac{5}{x^2+1}\,dx$$

$$= 2\ln(|x|) + 5\arctan(x) + C$$

18c)

$$\int \frac{2x^4+7x+1}{x^3+x^2-2}\,dx$$

$$(2x^4+0x^3+0x^2+7x+1):(x^3+x^2+0x-2)=2x-2$$
$$2x^4+2x^3+0x^2-4x$$
$$-2x^3+0x^2+11x+1$$
$$-2x^3-2x^2+0x+4$$
$$2x^2+11x-3$$

140 : 7 = 2
141 : 7 = 20 + $\frac{1}{7}$

$$\Rightarrow \frac{2x^4+7x+1}{x^3+x^2-2} = 2x-2+\frac{2x^2+11x-3}{x^3+x^2-2}$$

$x^3+x^2-2=0$ Teile von 2 here $\pm 1 \pm 2$

$1+1-2=0$ ✓

$$(x^3+x^2+0x-2):(x-1)=x^2+2x+2$$
$$x^3-x^2$$
$$2x^2+0x$$
$$2x^2-2x$$
$$2x-2$$
$$2x-2$$
$$0$$

$$x^2+2x+2=0$$
$$x_{1,2}=\frac{-2\pm\sqrt{4-4\cdot1\cdot2}}{2\cdot1}=\lightning$$

$$\Rightarrow \frac{2x^4+7x+1}{x^3+x^2-2} = 2x-2+\frac{2x^2+11x-3}{(x-1)(x^2+2x+2)}$$

$$\frac{2x^2+11x-3}{(x-1)(x^2+2x+2)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+2}$$

$$2x^2+11x-3 = A(x^2+2x+2) + (Bx+C)(x-1)$$

$$2x^2+11x-3 = Ax^2+2Ax+2A+Bx^2-Bx+Cx-C$$

$$2x^2+11x-3 = (A+B)x^2+(2A-B+C)x+2A-C$$

$$\begin{array}{rcl} A+B & = & 2 \\ 2A-B+C & = & 11 \\ 2A \quad -C & = & -3 \end{array} \quad \Big| \begin{array}{l} \\ \cdot 1 \\ \cdot 1 \end{array}$$

$$\begin{array}{rcl} 2A-B+C & = & 11 \\ 2A \quad -C & = & -3 \\ \hline 4A-B & = & 8 \\ A+B & = & 2 \\ \hline 5A & = & 10 \end{array}$$

$\rightsquigarrow A = 2 \rightsquigarrow B = 0$

$\rightsquigarrow 2 \cdot 2 - C = -3$

$C = 7$

$$\frac{2x^4+7x+1}{x^3+x^2-2} = 2x-2 + \frac{2}{x-1} + \frac{7}{x^2+2x+2}$$

$$\int f(x)\,dx = \int (2x-2)\,dx + \int \frac{2}{x-1}\,dx + \int \frac{7}{x^2+2x+2}\,dx$$

$$= x^2-2x+2\ln(|x-1|) + \int \frac{7}{x^2+2x+1^2-1^2+2}\,dx$$

$$= x^2 - 2x + 2\ln(|x-1|) + \int \frac{7}{(x+1)^2 + 1}\, dx$$

$$= x^2 - 2x + 2\ln(|x-1|) + 7\arctan(x+1) + C$$