19a)

$$\int \frac{14-2x}{(x-2)(x+3)}\,dx$$

$$\frac{14-2x}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}$$

$$14-2x = A(x+3) + B(x-2)$$

$x=-3$ einsetzen

$$20 = -5B$$

$$B = -4$$

$x=2$ einsetzen

$$10 = A \cdot 5$$

$$A = 2$$

$$\int f(x)\,dx = \int \frac{2}{x-2}\,dx + \int \frac{-4}{x+3}\,dx$$

$$= 2\ln(|x-2|) - 4\ln(|x+3|) + C$$

19c)

$$\int \frac{3x^2+x-3}{x^2(x-1)}\,dx$$

$$\frac{3x^2+x-3}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$3x^2+x-3 = Ax(x-1) + B(x-1) + Cx^2$$

$x=0$ einsetzen

$$-3 = -B$$

$$B = 3$$

$x=1$ einsetzen

$$1 = C$$

$x=2$, $B=3$, $C=1$ einsetzen

$$3\cdot 4 + 2 - 3 = A\cdot 2\cdot 1 + 3\cdot 1 + 1\cdot 4$$

$$11 = 2A + 7$$

$$4 = 2A$$

$$A = 2$$

$$\int f(x)\,dx = \int \frac{2}{x}\,dx + \int 3\cdot x^{-2}\,dx + \int \frac{1}{x-1}\,dx$$

$$= 2\ln(|x|) - 3x^{-1} + \ln(|x-1|)\,dx + C$$

20 a)

$$\int \frac{2x^2+9x+8}{x(x^2+2x+4)}\,dx$$

$x^2+2x+4=0$

$x_{1,2} = \frac{-2 \pm \sqrt{4-4\cdot 1\cdot 4}}{2\cdot 1}$ ↯

$$\frac{2x^2+9x+8}{x(x^2+2x+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+2x+4}$$

$$2x^2+9x+8 = Ax^2+2Ax+4A+Bx^2+Cx$$

$$2x^2+9x+8 = (A+B)x^2+(2A+C)x+(4A+B)$$

$A+B=2$

$2A+C=9$

$4A+B=8 \Rightarrow A=2$

$2+B=2 \Rightarrow B=0$

$2\cdot 2+C=9 \Rightarrow C=5$

$$\int f(x)\,dx = \int \frac{2}{x}\,dx + \int \frac{5}{x^2+2x+4}\,dx$$

$$= 2\ln(|x|) + \int \frac{5}{x^2+2x+1^2-1^2+4}\,dx$$

$$= 2\ln(|x|) + \int \frac{5}{(x+1)^2+3}\,dx$$

$$= 2\ln(|x|) + \int \frac{5}{3\left(\frac{1}{3}(x+1)^2+1\right)}\,dx$$

$$= 2\ln(|x|) + \frac{5}{3}\int \frac{1}{\left(\frac{1}{\sqrt{3}}(x+1)\right)^2+1}\,dx$$

$$= 2\ln(|x|) + \frac{5\sqrt{3}}{3}\arctan\left(\frac{1}{\sqrt{3}}(x+1)\right)+C$$

$$= 2\ln(|x|) + \frac{5\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}(x+1)\right)+C$$

20d)

$$\int \frac{8x+1}{4x^2+8x+8}\,dx = \qquad D = \sqrt{8^2-4\cdot 4\cdot 8}$$

$$= \int \frac{8x+1+7-7}{4x^2+8x+8}\,dx$$

$$= \int \frac{8x+8}{4x^2+8x+8}\,dx + \int \frac{-7}{4x^2+8x+8}\,dx$$

$$= \ln\left(4x^2+8x+8\right) + \int \frac{-7}{4(x^2+2x+2)}\,dx$$

$$= \ln\left(4x^2+8x+8\right) - \frac{7}{4}\int \frac{1}{x^2+2x+1^2-1^2+2}\,dx$$

$$= \ln\left(4x^2+8x+8\right) - \frac{7}{4}\int \frac{1}{(x+1)^2+1}\,dx$$

$$= \ln\left(4x^2+8x+8\right) - \frac{7}{4}\arctan(x+1)+C$$