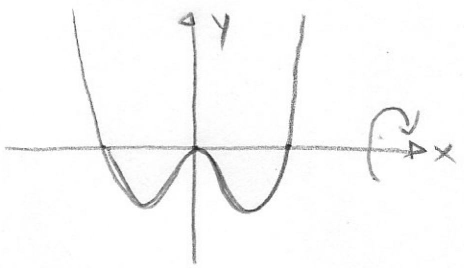
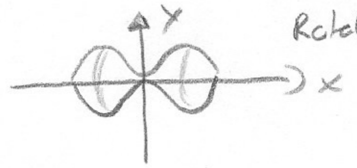
A4b

$f(x) = x^4 - x^2$

$f(x) = x^2(x^2-1)$

$x = 0$ doppelt $\quad x^2-1=0$, $x = \pm 1$





Rotationskörper

$V = \pi \int_{-1}^{1} f^2(x)\,dx = 2\pi \int_0^1 f^2(x)\,dx$

$= 2\pi \int_0^1 (x^4-x^2)^2\,dx =$

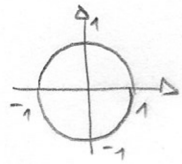
$= 2\pi \int_0^1 (x^8 - 2x^6 + x^4)\,dx = 2\pi \Big[\underbrace{\tfrac{1}{9}x^9 - \tfrac{2}{7}x^7 + \tfrac{1}{5}x^5}_{H}\Big]_0^1 \approx 0{,}167\,\text{FE}$

$= 2\pi\,(H(1) - H(0))$

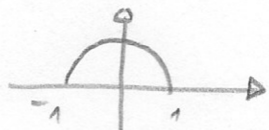
A4e) $f(x) = \sqrt{1-x^2}$ diese Funktion könnte man vielleicht gleich als "Kreis" erkennen

$y = \sqrt{1-x^2} \Rightarrow y^2 = 1-x^2$

$\Rightarrow x^2+y^2 = 1$



Somit ist $y = \sqrt{1-x^2}$ der obere Teil des Kreises



$V = \pi \int_{-1}^{1} f^2(x)\,dx = 2\pi \int_0^1 \left(\sqrt{1-x^2}\right)^2 dx$

$= 2\pi \int (1-x^2)\,dx = 2\pi \Big[\underbrace{x - \tfrac{1}{3}x^3}_{H}\Big]_0^1 = 2\pi\,(H(1) - H(0))$

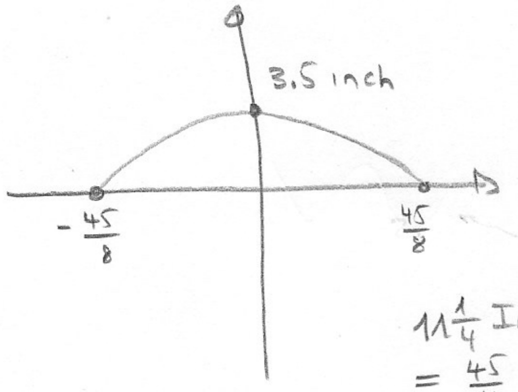
$= 2\pi\left(1 - \tfrac{1}{3}\right) = \tfrac{4}{3}\pi$

Es ist das Volumen einer Kugel mit Radius 1

$V = \tfrac{4}{3}\pi r^3 \underset{\text{mit } r=1}{\Rightarrow} V = \tfrac{4}{3}\pi \cdot 1^3 = \tfrac{4}{3}\pi$

A6

$$f(x) = ax^2 + b$$




$11\frac{1}{4}$ Inch $= \frac{45}{4}$ daraus folgt das Intervall $\left[-\frac{45}{8}, \frac{45}{8}\right]$

$$f(x) = ax^2 + 3.5$$

$$f\left(\frac{45}{8}\right) = 0$$

$$a \cdot \left(\frac{45}{8}\right)^2 + 3.5 = 0$$

$$a\left(\frac{45}{8}\right)^2 = -\frac{7}{2}$$

$$a = -\frac{224}{2025}$$

$$\Rightarrow f(x) = \frac{-224}{2025}x^2 + \frac{7}{2}$$

$$V = \pi \int_{-\frac{45}{8}}^{\frac{45}{8}} f^2(x)\,dx = 2\pi \int_0^{\frac{45}{8}} \left(-\frac{224}{2025}x^2 + \frac{7}{2}\right)^2 dx$$

$$= 2\pi \int_0^{45/8} \left(\frac{50176}{4100625}x^4 - \frac{1568}{2025}x^2 + \frac{49}{4}\right) dx$$

$$= 2\pi \left[\frac{50176}{20503125}x^5 - \frac{1568}{6075}x^3 + \frac{49}{4}x\right]_0^{45/8}$$

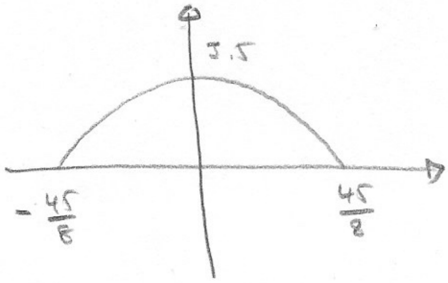
$$= 2\pi\left(H(45/8) - H(0)\right) = 2\pi\,\frac{147}{4} = \frac{147\pi}{2} \approx 230.91 \text{ Inch}^3$$

$$230.91 \text{ Inch}^3 \approx 3783.89 \text{ cm}^3$$

$$\approx 3.78 \text{ dm}^3$$

$$230.91 \text{ Inch}^3 \cdot 2.54^3 \frac{\text{cm}^3}{\text{Inch}^3} \approx 3783.89 \text{ cm}^3$$

A6 anderer Ansatz




man hatte den Ansatz für die Aufgabe folgendermassen machen können

$$f(x) = a\left(x + \frac{45}{8}\right)\left(x - \frac{45}{8}\right)$$

$$f(0) = 3.5$$

$$a \cdot \frac{45}{8} \cdot \left(-\frac{45}{8}\right) = 3.5$$

$$a = -\frac{224}{2025}$$

$$\Rightarrow f(x) = \frac{-224}{2025}\left(x + \frac{45}{8}\right)\left(x - \frac{45}{8}\right)$$

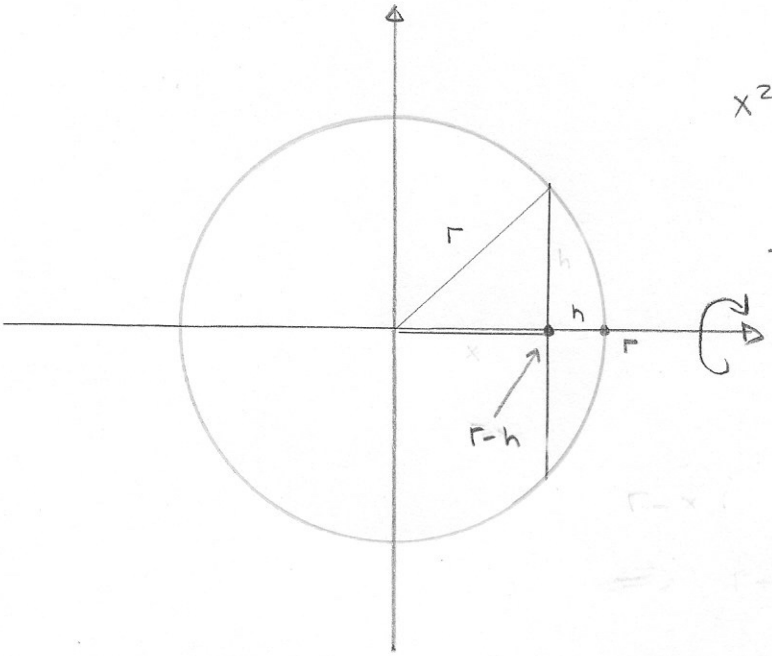
$$= -\frac{224}{2025}\left(x^2 - \left(\frac{45}{8}\right)^2\right) = \frac{-224}{2025}x^2 + \frac{7}{2}$$

Von hier aus geht es wie vorher

Aufg 7

Das Kugelkappenvolumen oder Kugelsegmentvolumen ist gem. Formelbuch S.10

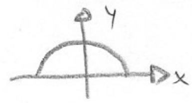
$$V = \frac{1}{3}\pi h^2 (3r-h)$$




$x^2 + y^2 = r^2$

$y = \pm\sqrt{r^2 - x^2}$

$\Rightarrow y = \sqrt{r^2 - x^2}$ nur der obere Teil




$$V = \pi \int_{r-h}^{r} f^2(x)\,dx = \pi \int_{r-h}^{r} \left(\sqrt{r^2 - x^2}\right)^2 dx = \pi \int_{r-h}^{r} (r^2 - x^2)\,dx$$

$$= \pi \left[ \overbrace{r^2 x - \frac{1}{3}x^3}^{H} \right]_{r-h}^{r} = \pi\left(H(r) - H(r-h)\right)$$

$$= \pi\left(r^3 - \frac{1}{3}r^3 - \left(r^2(r-h) - \frac{1}{3}(r-h)^3\right)\right)$$

$$= \pi\left(\frac{2}{3}r^3 - r^3 + r^2 h + \frac{1}{3}(r-h)^3\right)$$

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \end{array}$$

$$= \pi\left(-\frac{1}{3}r^3 + r^2 h + \frac{1}{3}\left(r^3 - 3r^2 h + 3rh^2 - h^3\right)\right)$$

$$= \pi\left(\cancel{-\frac{1}{3}r^3} \cancel{+ r^2 h} + \cancel{\frac{1}{3}r^3} - \cancel{r^2 h} + rh^2 - \frac{1}{3}h^3\right)$$

$$= \pi\left(rh^2 - \frac{1}{3}h^3\right) = \frac{1}{3}h^2\pi(3r-h) = \frac{1}{3}\pi h^2 (3r-h)$$