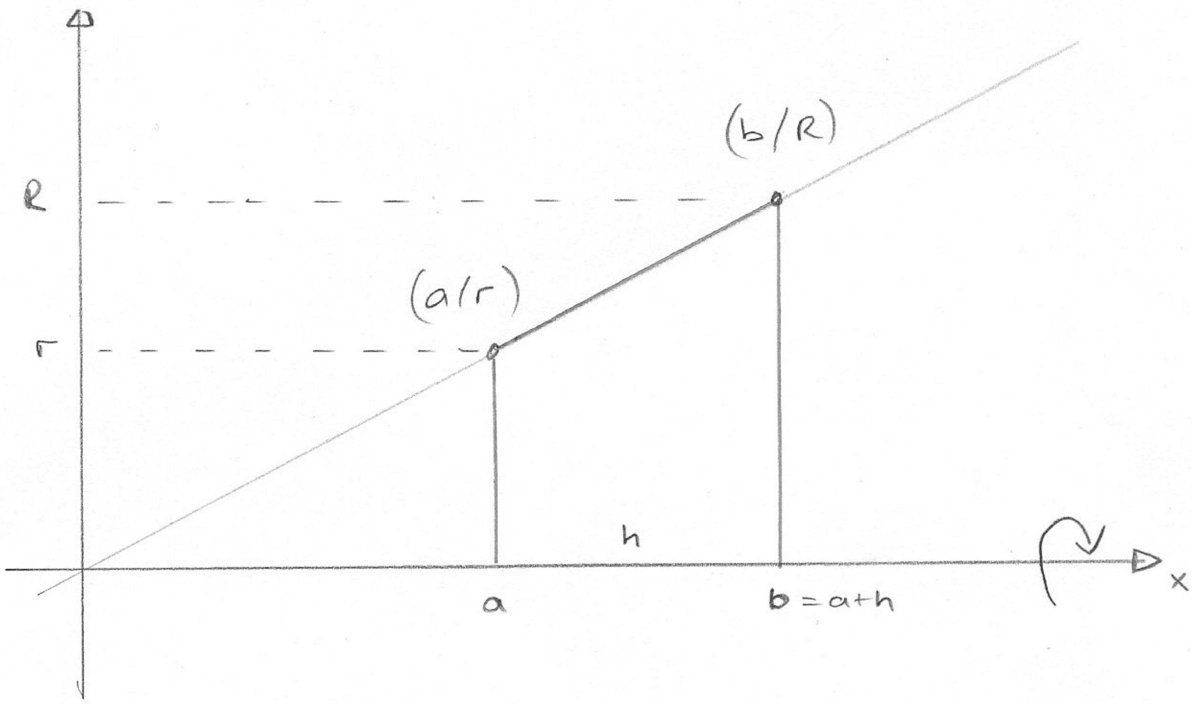
A8



$$f(x) = mx \qquad m = \frac{\Delta y}{\Delta x} = \frac{R-r}{b-a}$$

$$= \frac{R-r}{a+h-a} = \frac{R-r}{h}$$

a) Der Kegelstumpf ensteht durch Rotation eines Geradenstücks um die x-Achse. A8

b) $f(x) = mx$ Ansatz $P_1(a/r)$ $P_2(b/R)$ $b = a+h$

$$m = \frac{\Delta y}{\Delta x} = \frac{R-r}{b-a} = \frac{R-r}{a+h-a} = \frac{R-r}{h}$$

$$f(x) = \frac{R-r}{h} x$$

c) $f(a) = \frac{R-r}{h} a = r \implies a = \frac{rh}{R-r}$

$f(b) = \frac{R-r}{h} b = R \implies b = \frac{Rh}{R-r}$

d) $$V = \pi \int_a^b f^2(x)\,dx = \pi \int_{\frac{rh}{R-r}}^{\frac{Rh}{R-r}} \frac{(R-r)^2}{h^2} x^2\,dx$$

$$= \pi \left[\frac{(R-r)^2}{3h^2} x^3 \right]_{\frac{rh}{R-r}}^{\frac{Rh}{R-r}} =$$

$$= \pi \left(\frac{(R-r)^2}{3h^2} \frac{R^3h^3}{(R-r)^3} - \frac{(R-r)^2}{3h^2} \frac{r^3h^3}{(R-r)^3} \right)$$

$$= \frac{\pi h}{3} \left(\frac{R^3}{R-r} - \frac{r^3}{R-r} \right) = \frac{\pi}{3} h \left(\frac{R^3 - r^3}{R-r} \right)$$

$\frac{R^3 - r^3}{R-r} = ?$ Polynomdivision

$$(R^3 + 0R^2 + 0R - r^3) : (R-r) = R^2 + rR + r^2$$

$$\begin{array}{l}
R^3 - rR^2 \\
\hline
rR^2 + 0R \\
rR^2 - r^2R \\
\hline
r^2R - r^3 \\
r^2R - r^3 \\
\hline
0
\end{array}$$

$$\implies V = \frac{\pi}{3} h \left(R^2 + rR + r^2 \right)$$

a)

A12

$f(x) = a x^2$

$f(2) = a\, 2^2 = 3$

$a = \frac{3}{4}$

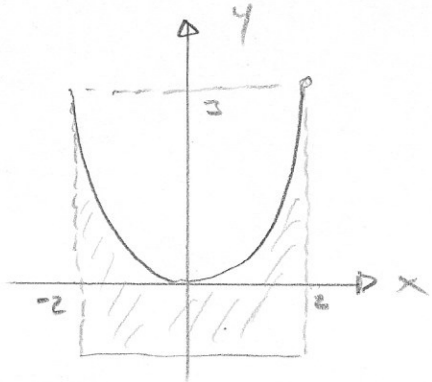
$y = \frac{3}{4}x^2$ Umkehrfunktion berechnen

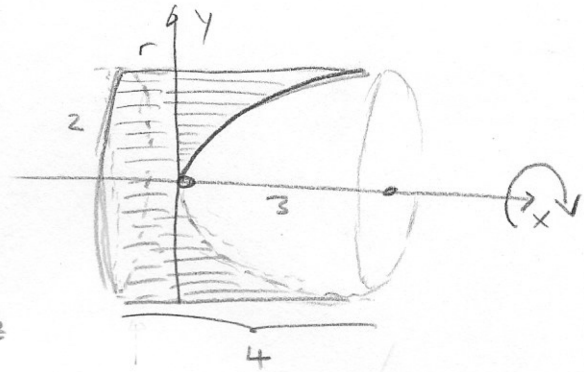
$\frac{4}{3}y = x^2 \quad | \sqrt{}$

$x = \pm\sqrt{\frac{4}{3}y} \quad / x \leftrightarrow y$

$y = \sqrt{\frac{4}{3}x}$

$\bar{f}(x) = \sqrt{\frac{4}{3}x}$



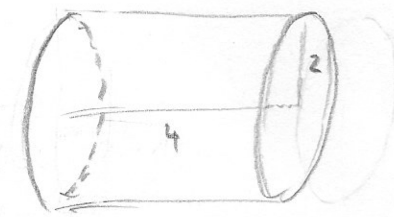


$V_{Totalzylinder} = \pi \cdot 2^2 \cdot 4 = 16\pi = V_{TZ}$

$V_{Paraboloid} = \pi \int_0^3 \bar{f}^2(x)\, dx$

$= \pi \int_0^3 \frac{4}{3}x\, dx = \pi \left[\frac{2}{3}x^2\right]_0^3$

$= 6\pi = V_P$



$V = \pi\, 2^2 \cdot 4$

$V = 16\pi$ VE

$V_{Flüssigkeit} = V_{TZ} - V_P$

$= 16\pi - 6\pi$

$= 10\pi$ VE

b)

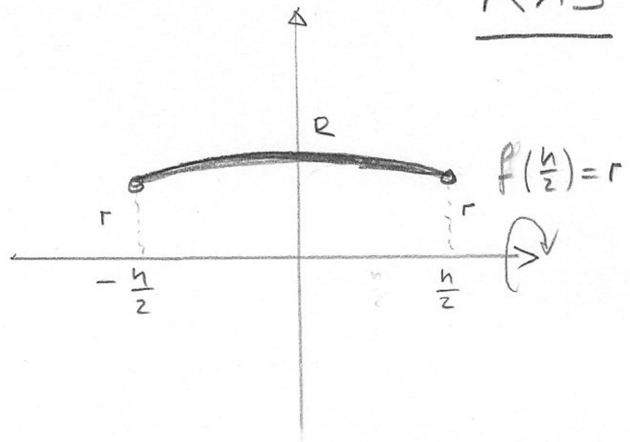
$V_{Bei\ Stillstand} = \cancel{\pi} \cdot 2^2 \cdot h = 10\cancel{\pi}$

$h = \frac{10}{4}$

$h = \frac{5}{2} = 2.5$ LE

$f(x) = ax^2 + R$ Ansatz

A13



$$f(\tfrac{h}{2}) = a\left(\tfrac{h}{2}\right)^2 + R = r$$

$$a = \frac{4(r-R)}{h^2}$$

$$f(x) = \frac{4(r-R)}{h^2}x^2 + R$$

$$V = 2\pi \int_0^{h/2} \left(\frac{4(r-R)}{h^2}x^2 + R \right)^2 dx$$

$$= 2\pi \int_0^{h/2} \left(\frac{16(r-R)^2}{h^4}x^4 + \frac{8\cdot(r-R)R}{h^2}x^2 + R^2 \right) dx$$

$$= 2\pi \left[\frac{16(r-R)^2}{5\,h^4}x^5 + \frac{8(r-R)R}{3\,h^2}x^3 + R^2x \right]_0^{h/2}$$

$$= 2\pi \left(\frac{16(r-R)^2}{5\,h^4}\frac{h^5}{32} + \frac{8(r-R)R}{3\,h^2}\frac{h^3}{8} + R^2\frac{h}{2} \right)$$

$$= 2\pi \left(\frac{(r-R)^2}{10}h + \frac{(r-R)Rh}{3} + R^2\frac{h}{2} \right)$$

$$= \pi h \left(\frac{(r-R)^2}{5} + \frac{2Rr - 2R^2}{3} + R^2 \right)$$

$$= \frac{\pi h}{15} \left(3(r^2 - 2rR + R^2) + 10Rr - 10R^2 + 15R^2 \right)$$

$$= \frac{\pi h}{15} \left(3r^2 - 6rR + 3R^2 + 10Rr - 10R^2 + 15R^2 \right)$$

$$= \frac{\pi h}{15} \left(8R^2 + 4Rr + 3r^2 \right) \qquad V = \frac{\pi h}{15}(8r^2 + 4r^2 + 3r^2)$$

$$V = \frac{\pi h}{\cancel{15}} \cdot \cancel{15} r^2 = \pi h r^2 = \pi r^2 h$$